INDIAN STATISTICAL INSTITUTE
CHENNAI CENTRE
M.STAT First Year
2015-16 Semester II

Large Sample Statistical Methods
Back Paper Examination

Points for each question is in brackets. Total Points 100.
Students are allowed to bring 4 pages (front and back) of hand-written notes
Duration: 3 hours

1. (15) If F is discrete, then show that $F_n \Rightarrow F$ iff for each x in the support of F, $P(X_n = x) \to P(X = x)$.

2. (10) Let $X_i, i = 1, 2, \cdots, n$ be independent Bernoulli(1/2) random variables and let $T_n = \frac{1}{n}\sum_{i=1}^n X_i$. Show that $4n\left[\frac{1}{4} - T_n(1 - T_n)\right]$ has asymptotic χ_1^2 distribution.

3. (15) Show that the Pearson chi-square statistic is a Wald statistic. Hence derive its asymptotic distribution under the null hypothesis.

4. (15) Let $X_1, \cdots, X_n$ be iid $N(0, \sigma^2)$. Find the asymptotic relative efficiency of the estimator $\delta_n = \sqrt{\frac{\pi}{2}}\sum \mid X_i \mid /n$ with respect to the MLE$=\sqrt{\sum X_i^2/n}$ of σ

5. (10) Use the asymptotic joint distribution of sample quantiles to derive the asymptotic distribution of the interquartile range.

6. (20) Let $X_1, \cdots, X_n$ be iid according to the logistic distribution with cdf

$$F_\theta(x) = \frac{1}{1 + e^{-(x-\theta)}}$$

   (a) Show that the likelihood equation has unique root $\hat{\theta}_n$ that maximizes the likelihood function.

   (b) Find the asymptotic distribution of $\hat{\theta}_n$.

   (c) Show that $\overline{X}_n$ is a consistent estimator of θ.

   (d) Suggest an estimator that can be computed explicitly and has the same asymptotic distribution as $\hat{\theta}_n$.

7. (15) Consider a symmetric kernel $h(x_1, \cdots, x_m)$ satisfying $E(h^2) < \infty$. Define $h_c(x_1, \cdots, x_c) = \mathrm{E}h(x_1, \cdots, x_c, X_{c+1}, \cdots, X_m)$ for $1 \le c \le m$ and $\zeta_c = \mathrm{Var}h_c(X_1, \cdots, X_c)$.

   (a) Show that $0 \le \zeta_1 \le \zeta_2 \le \cdots \le \zeta_m$.

   (b) Show that $\zeta_1 \le \frac{1}{2}\zeta_2$